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INFLUENCE OF FLUID VISCOSITY AND PRESSURE ON SHELL VIBRATIONS IN A FLUID*

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It is shown that for a broad class of problems in shell theory the fluid viscosity can be taken into account by separating the system of viscous fluid equations into two subsystems, one of which is integrated in quadratures by taking account of the viscosity, while the other agrees with the Helmholtz equation. Consequently, the related problem for a shell making contact with a fluid is reduced to integration of the equations of shell theory with certain additional terms in combination with the Helmholtz equation. It is clarified that the hydrostatic fluid pressure only affects the resonance frequencies and the amplitudes of those vibrations that satisfy the definite conditions formulated below.

1. The influence of fluid viscosity and hydrostatic pressure will be investigated separately for clarity.

We introduce a curvilinear coordinate system $\alpha_1, \alpha_2, \alpha_3$ in such a way that the coordinate lines α_1 and α_2 agree on the shell middle surface $\alpha_3 = 0$ with the lines of curvature, and the α_3- lines are orthogonal to them. Later, to within the error allowable in shell theory, we will consider the shell surface in contact with the fluid to coincide with the shell middle surface $\alpha_3 = 0$ and the lines α_3 to be straight near the shell.

The equations of viscous fluid motion, neglecting non-linear terms, have the form

$$\begin{aligned} p_k &= -p - i2\mu\omega\nabla_k v_k + i\cdot{}^2/_3\mu\omega \operatorname{div} \mathbf{v} \\ p_{kl} &= -i\mu\omega(\nabla_k v_l + \nabla_l v_k) \\ \nabla_k p &= \rho_0\omega^2 v_k - i\mu\omega\Delta v_k - i\,{}^1/_3\mu\omega\nabla_k \operatorname{div} \mathbf{v} \\ \rho_0 \operatorname{div} \mathbf{v} &+ c_0^{-2} p = 0 \end{aligned} \tag{1.1}$$

Here v_k are components of the displacement vector of the fluid particles, p_k and p_{kl} are stress tensor components, p is the sound pressure, μ is the fluid viscosity, ρ_0 is the fluid density, and c_0 is the speed of sound in the fluid. It is assumed that the fluid vibrations are caused by vibrations of the shell which have the form $e^{-i\omega t}$, where ω is the angular frequency, Δ is the Laplace operator in curvilinear coordinates, and the notation $\nabla_k v_l$, $\nabla_k p$ is given by the following formulas:

$$\nabla_k v_l = \frac{1}{H_k}\frac{\partial v_l}{\partial \alpha_k} - \frac{v_k}{H_l}\frac{\partial H_k}{\partial \alpha_l} + \delta_k{}^l \sum_{j=1}^{3} \frac{v_j}{H_j}\frac{\partial \ln H_l}{\partial \alpha_j}$$

$$\nabla_k p = \frac{1}{H_k}\frac{\partial p}{\partial \alpha_k}, \quad \delta_k{}^l = \begin{cases} 1, & k = l \\ 0, & k \neq l \end{cases}, \quad H_3 = 1$$

We use the small parameter η of shell theory, equal to the shell relative half-thickness, for an asymptotic analysis of the fluid equations. We will assume the variability of the desired fluid quantities in the coordinates α_1, α_2 to be the same near the shell as the desired shell-theory quantities

$$\frac{\partial}{\partial \alpha_n} = \frac{\eta^{-s}}{R}\frac{\partial}{\partial \xi_n} \tag{1.2}$$

The subscripts n, m take the values 1, 2 everywhere while the subscripts k, l take the values 1, 2, 3, and R is the characteristic dimension of the shell. The index of variability s is found in such a manner that differentiation with respect to the dimensionless variables ξ_l does not result in a substantial increase or decrease in the desired functions.

Moreover, we introduce the dimensionless frequency parameter λ by the following formula

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$$\omega^2 R^2 \rho_1 / E = \lambda \eta^{2r} \tag{1.3}$$

The power $2r$ is selected so that the quantity λ is commensurate with unity, and ρ_1, E are the shell density and elastic modulus.

The conditions that the shell and fluid displacement vectors be equal should be satisfied on the shell surface in contact with the fluid

$$v_n = u_n, \quad v_3 = -w \tag{1.4}$$

(u_n, w are the shell displacements). The equations and notation utilized below agree with those used in shell theory /1/. To be specific we will assume the fluid to be outside the shell.

2. As is shown below, the displacement and stress field described by the system of equations of fluid motion (1.1) can be separated into two fields described by simpler systems of equations.

We will agree to call the first of these fields damped. For vibrations of this kind, the fluid particle displacements v_1, v_2 tangential to the fluid surface are substantially greater than the normal displacements v_3. These vibrations damp out rapidly with distance from the shell. The following asymptotic form, that does not result in contradictions

$$\begin{aligned} &v_n/R = \eta^0 v_{n*}, \quad v_3/R = \eta^{a-s-r/2} v_{3*} \\ &p_{n3}/(\rho_0\omega^2 R^2) = \eta^{a-r/2} p_{n3*} \\ &(p, p_{nm}, p_n, p_3)/(\rho_0\omega^2 R^2) = \eta^{2a-s-r}(p_*, p_{nm*}, p_{n*}, p_{3*}) \\ &\mu/(\rho_0 R c_1) = A\eta^{2a}, \ c_1^2/c_0^2 = B\eta^{-2b}, \ \alpha_3 = \eta^{a-r/2} Rz \end{aligned} \tag{2.1}$$

should be taken for the desired magnitudes of the quasitangential fluid vibrations.

As is customary in the asymptotic method, the exponents of η should be defined so that the dimensionless quantities (with the asterisks) are of the same order. The numbers A, B are of the order of one, and c_1 is the speed of sound in the shell material.

We make the change of variables (1.2), we substitute (2.1) and (1.3) into (1.1) and consequently we obtain the equations

$$\begin{aligned} &\eta^{2a-2s-r}\nabla_{n*}p_* = v_{n*} - i(A/\sqrt{\lambda})[\partial^2 v_{n*}/\partial z^2 + \\ &\quad \eta^{2a-2s-r}\nabla_{12*}v_{n*}] + iAB\sqrt{\lambda}\eta^{2a-2b-r}p_*/3 \\ &p_{n3*} = -i(\partial v_{n*}/\partial z + \eta^{2a-2s-r}\nabla_{n*}v_{3*}) \\ &\partial v_{3*}/\partial z = -\nabla_1 v_{1*} - \nabla_2 v_{2*} - \lambda B\eta^{2a-2b+r}p_* \\ &\partial p_*/\partial z = v_{3*} - i(A/\sqrt{\lambda})[\partial^2 v_{3*}/\partial z^2 + \eta^{2a-2s-r}\nabla_{12*}v_{3*}] + \\ &\quad iA\sqrt{\lambda}B\eta^{2a-2b+r}\partial p_*/\partial z \\ &p_{nm*} = -i(A/\sqrt{\lambda})(\nabla_{n*}v_{m*} + \nabla_{m*}v_{n*}) \\ &p_{n*} = -p_* - 2i(A/\sqrt{\lambda})\nabla_{n*}v_{n*} - 2iAB\sqrt{\lambda}\eta^{2a-2b-r}p_*/3 \\ &p_{3*} = -p_* - 2i(A/\sqrt{\lambda})\partial v_{3*}/\partial z - 2iAB\sqrt{\lambda}\eta^{2a-2b-r}p_*/3 \end{aligned} \tag{2.2}$$

$$\begin{aligned} &\nabla_{n*}v_{m*} = \frac{1}{H_n}\frac{\partial v_{n*}}{\partial \xi_n} - R\eta^s\frac{1}{H_m}\frac{\partial H_n}{\partial \alpha_m} + R\eta^s\delta_n{}^m\sum_{j=1}^{3}\frac{v_j}{H_j}\frac{\partial \ln H_n}{\partial \alpha_j} \\ &\nabla_{12*} = \frac{1}{H_1H_2}\left[\frac{\partial}{\partial \xi_1}\left(\frac{H_2}{H_1}\frac{\partial}{\partial \xi_1}\right) + \frac{\partial}{\partial \xi_2}\left(\frac{H_1}{H_2}\frac{\partial}{\partial \xi_2}\right)\right] \\ &\nabla_{n*}p_* = \frac{1}{H_n}\frac{\partial p_*}{\partial \xi_n} \end{aligned}$$

Discarding small terms of the order of $(\eta^{2a-2s-r} + \eta^{2a-2b+r})$, we obtain a simplified system of equations in which we note the desired quantities by the superscript (1):

$$\begin{aligned} &\frac{\partial^2 v_n^{(1)}}{\partial \alpha_3^2} + i\frac{\rho_0\omega}{\mu}v_n^{(1)} = 0 \\ &p_{n3}^{(1)} = -i\mu\omega\frac{\partial v_n^{(1)}}{\partial \alpha_3}, \quad \frac{\partial v_3^{(1)}}{\partial \alpha_3} = -\nabla_1 v_1^{(1)} - \nabla_2 v_2^{(1)} \\ &\frac{\partial p^{(1)}}{\partial \alpha_3} = \rho_0\omega^2 v_3^{(1)} - i\mu\omega\frac{\partial^2 v_3^{(1)}}{\partial \alpha_3}, \quad p_{nm}^{(1)} = -i\mu\omega(\nabla_n v_m^{(1)} + \nabla_m v_n^{(1)}) \\ &p_n^{(1)} = -p^{(1)} - 2i\mu\omega\nabla_n v_n^{(1)}, \quad p_3^{(1)} = -p^{(1)} - 2i\mu\omega\frac{\partial v_3^{(1)}}{\partial \alpha_3} \end{aligned} \tag{2.3}$$

We note that the fluid can be considered incompressible to the Customary accuracy for quasitangential vibrations.

The solution of (2.3) has the following form (we present only those quantities that are required later):

$$v_n^{(1)} = f_n e^{(i-1)\delta\alpha_3}, \quad p_{n3}^{(1)} = (1+i)\mu\omega\delta f_n e^{(i-1)\delta\alpha_3} \tag{2.4}$$
$$v_3^{(1)} = \frac{1+i}{2\delta} e^{(i-1)\delta\alpha_3}(\nabla_1 f_1 + \nabla_2 f_2), \quad \delta = \frac{\rho_0\omega}{2\mu}$$

Here f_n are arbitrary functions that enable the conditions that the tangential shell and fluid displacements be equal on the contact surface to be satisfied.

An iteration process can be constructed to refine the desired magnitudes of the damped solution. After the first approximation has been found, the magnitudes of the next, second, approximation are found from the system

$$\frac{\partial^2 v_n^{(2)}}{\partial \alpha_3^2} + i\frac{\rho_0\omega}{\mu} v_n^{(2)} = -\frac{i}{\mu\omega}\nabla_n p^{(1)} + \frac{1}{3}\nabla_n \operatorname{div} v^{(1)} - \nabla_{12} v_n^{(1)} \tag{2.5}$$
$$p_{n3}^{(2)} + i\mu\omega\frac{\partial v_n^{(2)}}{\partial \alpha_3} = -i\mu\omega\nabla_n v_3^{(1)}$$
$$\frac{\partial v_3^{(2)}}{\partial \alpha_3} + \nabla_1 v_1^{(2)} + \nabla_2 v_2^{(2)} = -\frac{1}{\rho_0 c_0^2} p^{(1)}$$
$$\frac{\partial p^{(2)}}{\partial \alpha_3} - \rho_0\omega^2 v_3^{(2)} + i\mu\omega\frac{\partial^2 v_3^{(2)}}{\partial \alpha_3^2} = -i\mu\omega\nabla_{12} v_3^{(1)} + \frac{\mu\omega^3}{3\rho_0 c_0^2}\frac{\partial p^{(1)}}{\partial \alpha_3}$$
$$p_{nm}^{(2)} + i\mu\omega(\nabla_n v_m^{(2)} + \nabla_m v_n^{(2)}) = 0$$
$$p_n^{(2)} + p^{(2)} + 2i\mu\omega\nabla_n v_n^{(2)} = -i\frac{2\mu\omega}{3\rho_0 c_0^2} p^{(1)}$$
$$p_3^{(2)} + p^{(2)} + 2i\mu\omega\frac{\partial v_3^{(2)}}{\partial \alpha_3} = -i\frac{2\mu\omega}{3\rho_0 c_0^2} p^{(1)}$$
$$\nabla_{12} = \frac{1}{H_1 H_2}\left[\frac{\partial}{\partial \alpha_1}\left(\frac{H_2}{H_1}\frac{\partial}{\partial \alpha_1}\right) + \frac{\partial}{\partial \alpha_2}\left(\frac{H_1}{H_2}\frac{\partial}{\partial \alpha_2}\right)\right]$$

Formulas (2.5) have the same structure as (2.3) in the sense that the next approximation differs from the preceding one by just terms with lower subscripts, i.e., by known free terms.

The second of the fields comprising the total fluid displacement and stress field should satisfy the condition of equality of the displacements normal to the contact surface. We agree to call it penetrating. We introduce its fluid potential φ by the following formulas:

$$v_n = -\frac{1}{H_n}\frac{\partial \varphi}{\partial \alpha_n}, \quad v_3 = -\frac{\partial \varphi}{\partial \alpha_3}, \quad p = -\rho_0\omega^2\varphi + \frac{4}{3} i\mu\omega\Delta\varphi \tag{2.6}$$

The asymptotic form of the principal desired magnitudes of the penetrating fluid vibrations agrees with the asymptotic form presented in /2/ for an ideal fluid. We write down the asymptotic representation of the desired fluid quantities

$$\varphi/R^2 = \eta^q \varphi_*, \quad v_n/R = \eta^{q-s} v_{n*}, \quad v_3/R = \eta^0 v_{3*} \tag{2.7}$$
$$(p, p_n, p_3)/(\rho_0\omega^2 R^2) = \eta^q (p_*, p_{n*}, p_{3*})$$
$$p_{n3}/(\rho_0\omega^2 R^2) = \eta^{2a-r-s} p_{n3*}, \quad p_{nm}/(\rho_0\omega^2 R^2) = \eta^{2a-2s-r+q} p_{nm*}$$
$$\alpha_3 = \eta^q R z, \ q = \max(s, \ b-r)$$

Taking account of (2.6) and (2.7), we can write (1.1) in asymptotic form. The terms taking account of the viscosity in the equations of the fluid penetrating vibrations are $O(\eta^{2a-2s-r} + \eta^{2a-2b+r})$ as compared with the principal terms, while they are much greater $O(\eta^{a-r/2} + \eta^{a-s-r/2})$, in the fluid damped-vibrations equations, consequently to a first approximation the viscosity should be retained only in the fluid damped-vibrations equations. In this case to a first approximation the fluid penetrating vibrations will be described by the Helmholtz equation for an ideal fluid

$$\Delta\varphi^{(1)} + \frac{\omega^2}{c_0^2}\varphi^{(1)} = 0, \quad v_n^{(1)} = -\nabla_n\varphi^{(1)}, \quad v_3^{(1)} = -\frac{\partial \varphi^{(1)}}{\partial \alpha_3} \tag{2.8}$$
$$p^{(1)} = -\omega^2\rho_0\varphi^{(1)}, \quad p_k^{(1)} = p^{(1)}, \quad p_{n3}^{(1)} = 0$$

The defined desired quantities of the second approximation can be determined from the following recursion system in whose right sides are the known quantities found from (2.8)

$$\Delta\varphi^{(2)} + \frac{\omega^2}{c_0^2}\varphi^{(2)} = -i\frac{4\mu\omega^3}{3\rho_0 c_0^4}\varphi^{(1)}$$
$$p^{(2)} + \rho_0\omega^2\varphi^{(2)} = i\frac{4}{3}\mu\omega\Delta\varphi^{(1)}$$
$$v_n^{(2)} + \frac{1}{H_n}\frac{\partial \varphi^{(2)}}{\partial \alpha_n} = 0, \quad v_3^{(2)} + \frac{\partial \varphi^{(2)}}{\partial \alpha_3} = 0$$

$$p_k^{(2)} + p^{(2)} = -i \cdot 2\mu\omega \nabla_k v_k^{(1)} + i\frac{2}{3}\mu\omega \operatorname{div} \mathbf{v}^{(1)}$$

$$p_{kl}^{(2)} + i\mu\omega(\nabla_k v_l^{(2)} + \nabla_l v_k^{(2)}) = 0$$

3. We write down the complete system of equations describing the motion of a shell making contact with a viscous fluid. We will use the method proposed in Sect.2 to separate the complete fluid stress and displacement field into two component fields by using the simplified formulas obtained above, to describe the fluid motion.

The shell equilibrium equations are

$$\frac{1}{A_n}\frac{\partial T_n}{\partial \alpha_n} - \frac{1}{A_m}\frac{\partial S_m}{\partial \alpha_m} + k_m(T_n - T_m) + k_n(S_n - S_m) - \frac{N_n}{R_n} + 2h\rho_1\omega^2 u_n + p_{n3}|_{\alpha_3=\alpha_{3*}} + X_n = 0 \tag{3.1}$$

$$\frac{T_1}{R_1} + \frac{T_2}{R_2} + \frac{1}{A_1}\frac{\partial N_1}{\partial \alpha_1} + \frac{1}{A_2}\frac{\partial N_2}{\partial \alpha_2} + k_2 N_1 + k_1 N_2 + 2h\rho_1\omega^2 w + \rho_0\omega^2\varphi|_{\alpha_3=\alpha_{3*}} + Z = 0$$

$$\frac{1}{A_n}\frac{\partial G_n}{\partial \alpha_n} + \frac{1}{A_m}\frac{\partial H_m}{\partial \alpha_m} + k_m(G_n - G_m) - k_n(H_n - H_m) - N_n = 0$$

$$k_n = \frac{1}{A_n A_m}\frac{\partial A_n}{\partial \alpha_m}$$

The non-penetration conditions are

$$\left(\frac{\partial \varphi}{\partial \alpha_3} + v_3\right)\Big|_{\alpha_3=\alpha_{3*}} = -w \tag{3.2}$$

Here p_{n3} and v_3 are defined by the formulas

$$p_{n3}|_{\alpha_3=\alpha_{3*}} = (1+i)\mu\omega\left(u_n + \frac{1}{A_n}\frac{\partial \varphi}{\partial \alpha_n}\right)$$

$$v_3|_{\alpha_3=\alpha_{3*}} = \frac{1+i}{2\delta}\sum_{j=1}^{2}\frac{1}{A_j}\frac{\partial}{\partial \alpha_j}\left(u_j + \frac{1}{A_j}\frac{\partial \varphi}{\partial \alpha_j}\right)$$

To obtain a closed system, the Helmholtz Eq.(2.8), the elasticity relationships and the strain-displacement formulas /1/ should be added to (3.1) and (3.2).

By investigating this system by an asymptotic method it can be shown that the fluid viscosity makes the greatest contribution, as might have been expected, to the quasitangential shell vibrations. In this case the terms taking account of the viscosity are of the order of $(\eta^{a+d-1+s/2} + \eta^{a+d-1+b-s/2})$ for $s \leqslant 1-d-b$ and $O(\eta^{a+d-1+s/2} + \eta^{a-3s/2})$ for $s > 1-d-b$ as compared with the principal terms, where d is determined from the following formula: $\rho_1/\rho_0 = \eta^{-d}$.

As an illustration we will find the resonance frequencies of a circular cylindrical shell filled with a viscous fluid. We will assume that a shell of length L $(L/R = l)$, radius R and thickness $2h$ performs axisymmetric quasitangential vibrations. If the fluid is ideal, the natural frequencies of the quasitangential shell vibrations are approximately equal to the natural frequencies of vibration of a shell without fluid. The equation of axisymmetric longitudinal vibrations in this case has the form

$$\frac{1}{R^2}\frac{d^2u}{d\xi^2} + \lambda u = 0, \quad \lambda = \frac{\omega^2\rho_1 R^2}{E}$$

(u is the longitudinal displacement and ξ is the longitudinal coordinate).

In the case under consideration, for a viscous fluid we obtain the preceding equation from (3.1), in which

$$\lambda = \frac{\omega^2\rho_1 R^2}{E}(1 + \delta + i\delta), \quad \delta = \frac{1}{2h\rho_1}\sqrt{\frac{\rho_0\mu}{2\omega}}$$

The results of calculating of the dimensionless quantities $\omega_* = \omega 2lR\sqrt{\rho_1/E}/\pi$, proportional to the natural frequencies ω are presented in Table 1: a) for a steel shell without fluid (they approximately equal the natural frequencies of a fluid-filled shell); b) for a shell making contact with water at $0°$C ($\mu = 0.0017$ kg/m·sec, and $\rho_0 = 10^3$ kg/m^3); c) for a shell making contact with glycerine at $3°$C ($\mu = 4.2$ kg/m·sec and $\rho_0 = 1.265$ kg/m^3).

4. We will investigate the influence of hydrostatic pressure on shell vibrations in a fluid. It has been shown /3/ that a preliminary static load influences the free shell vibrations in a vacuum only when: a) the value of the static load is of the same order as the

critical load (i.e., the load which causes buckling within the framework of linear theory); b) the shell vibrations mode should be similar to the buckling mode under the effect of this static load.

Table 1

a	b		c	
	$h = 5$ mm	1 mm	5 mm	1 mm
5	4.98 − i0.0186	4.91 − i0.0903	4.03 − i0.677	2.07 − i0.898
25	24.9 − i0.0418	24.8 − i0.206	22.7 − i1.90	15.7 − i4.68
45	44.9 − i0.0561	44.7 − i0.278	42.0 − i2.67	31.5 − i7.78

Table 2

$\frac{p_0}{p}$	$l = 20$, $n = 2$	2, 6	1, 9	1, 20
1.5	0.737	0.600	0.650	0.918
2	0.815	0.725	0.753	0.938
3	0.880	0.826	0.846	0.960

The validity of the rule formulated above is confirmed by a numerical computation carried out for a circular cylindrical shell submerged in water. The shell executes forced vibrations under the effect of a normal dynamical surface load changing according to the law $A \cos n\beta \sin (m\pi\xi/l)\, e^{-i\omega t}$ (β is the circumferential coordinate and ξ is the longitudinal coordinate). The relative shell thickness is 0.01, the radius is $R = 1$ m and the length is $L = lR$.

The initial equations of the shell-fluid system differ from those presented in /4/ just by a team taking account of the hydrostatic pressure in the third equilibrium equation of shell theory and equal to $T_\beta^{(s)}\varkappa_2$, where $T_\beta^{(s)}$ is determined from the solution of the static problem (a cylindrical shell subjected to hydrostatic pressure), $\varkappa_2$ is the bending strain of the β-line. We consider the shell hinge-supported and enclosed in a stiff screen.

To determine the pressure of the far sound field we use the usual method of solving this problem (see /4/, for example): we perform a Fourier integral transform in the longitudinal coordinate ξ, whereupon we obtain a system of equations with constant coefficients for the shell and the Bessel equation for the fluid. We find the second pressure far from the shell by applying an inverse Fourier transform to the solution and calculating the integral obtained by the stationary-phase method.

We will compare the resonance frequencies found for shells without taking account of the hydrostatic pressure (denoted by ω_0) and with the hydrostatic pressure taken into account (we denote the appropriate resonance frequencies by ω_p).

The ratios ω_p/ω_0 are presented in Table 2 for different values of $p_0/p, l, n$ (p is the hydrostatic pressure acting on the shell, and p_0 is the critical pressure). For the buckling mode we have $m = 1$, while the value of n depends on the shell length /5/ for fixed thickness and radius. The first three colums of Table 2 show numbers characterizing the vibrations that agree in mode with the buckling mode; consequently, the resonance frequencies for the case when the hydrostatic pressure is 1.5 times less than the critical were reduced by 30-40% compared with the resonance frequencies found without taking account of the hydrostatic pressure. The fourth column gives analogous data for shell whose mode of vibration differs from the buckling mode, consequently, the influence of the hydrostatic pressure is negligible.

Thus, a method of separation is proposed that enables substantial simplification of the integration of (1.1) to be achieved for a viscous fluid interacting with an elastic shell. The method is not suitable only in those cases when a certain factor dependent on the properties of the shell material and the fluid, is commensurate with unity. This holds only for shells with a small modulus of elasticity and a fluid with high viscosity. It is shown that for shells submerged in the fluid, the order of magnitude of the hydrostatic pressure affecting definite modes of the shell vibrations can be indicated without solving the problem.

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STATE OF STRESS AND STRAIN OF A SMALL NEIGHBOURHOOD OF THE APEX OF A WEDGE FOR A PHYSICAL NON-LINEARITY AND DIFFERENT BOUNDARY CONDITIONS*

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Three plane strain problems of a small neighbourhood of the apex of a physically non-linear incompressible wedge are investigated by the Cherepanov-Rice-Hutchinson method using a non-linear differential equation for the Airy stress function: Problem 1 - one face is free, and a smooth contact condition is given on the other; Problem 2 - one face if free, and a flexible inextensible cover plate is glued to the other; Problem 3 - one fact is free and the condition of adhesion to a stiff flat stamp is given on the other. Numerical results are presented that illustrate the influence of the degree of non-linearity of the governing relationships and the wedge aperture angle on the solution. The method is also applied to the stream function which enables us to formulate an analogy between different plane problems and affords the possibility of extending it to the axisymmetric case. In many problems of the mechanics of a deformable solid, the investigation of the asymptotic form of the solution near an angular point of a domain occupied by a body plays a fundamental role. In the elastic case this question has been studied quite broadly and an extensive literature exists. The situation is more complicated if the governing relationships are non-linear. The majority of papers deal only with the case of a crack. This paper attempts to fill this gap somewhat.

1. We consider the problem of the equilibrium of a wedge with apperture angle α from a material subjected to the law

$$\varepsilon_u = A\sigma_u{}^m,\quad \varepsilon_{kk}=0,\quad S_{ij}=\sigma_u\varepsilon_u^{-1}\varepsilon_{ij},\quad A,m=\text{const},\quad m\geqslant 1 \tag{1.1}$$

$$\sigma_u = 6^{-1/2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+6\sigma_{12}{}^2+6\sigma_{23}{}^2+6\sigma_{31}{}^2]^{1/2}$$

$$\varepsilon_u = 6^{-1/2}[(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2+6\varepsilon_{12}{}^2+6\varepsilon_{23}{}^2+6\varepsilon_{31}{}^2]^{1/2}$$

Here S_{ij} are the components of the stress deviator in a certain orthonormalized basis, σ_u is the stress intensity, ε_{ij} are the components of the strain or strain rate tensor depending on the specific model: if the problem of non-linear steady creep is considered, then ε_{ij} is the rate, if an elastic-plastic tension-compression diagram is described by (1.1), generally speaking, then ε_{ij} are tensor components of small strain. There is no need to make the physical meaning of ε_{ij} specific; by virtue of a well-known analogy the fundamental equations are written identically, and consequently, we will henceforth call ε_{ij} the strain for brevity, and ε_u the strain intensity.

We assume the strain to be planar. In polar coordinates with centre of the wedge apex we have

$$S_r=-S_\vartheta=\frac{\sigma_u}{\varepsilon_u}\varepsilon_r,\quad S_{r\vartheta}=\frac{\sigma_u}{\varepsilon_u}\varepsilon_{r\vartheta},\quad \sigma=\sigma_z=\frac{\sigma_r+\sigma_\vartheta}{2} \tag{1.2}$$

$$\sigma_u=[{}^1/_4(\sigma_r-\sigma_\vartheta)^2+\sigma_{r\vartheta}^2]^{1/2},\quad \varepsilon_u=[\varepsilon_r{}^2+\varepsilon_{r\vartheta}^2]^{1/2}$$

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